Random Graals 2008

Organizers: Alan Frieze, Alessandro Pancones, Eli Upfal (Chair)

ABSTRACTS

Listed in alphabetical order by last name

The Triangle-free Process.

Tomas Bohman - CMU

Consider the following random graph process. We begin with the empty graph on n vertices and add edges chosen at random one at a time. Each edge is chosen uniformly at random from the collection of pairs of vertices that do not form triangles when added as edges to the existing graph. In this talk I discuss an analysis of the triangle-free process using the so-called differential equations method for random graph processes. It turns out that with high probability the triangle-free process produces a Ramsey R(3,t) graph, a triangle-free graph whose independence number is within a multiplicative constant factor of the smallest possible.

Algorithmic barriers from phase transitions

Amin Coja-Oghlan – University of Edimburgh

For many random Constraint Satisfaction Problems, we have reasonably good estimates of the largest constraint density for which solutions exist. These estimates are obtained by non-constructive arguments such as the "second moment method". In most cases no polynomial-time algorithm is known to solve instances of these problems even at much smaller densities than the threshold for the existence of solutions. In effect, random problem instances have been considered prime examples of "hard" instances. For example, it is very easy to color a random graph using twice as many colors as its chromatic number. Yet, to date, no algorithm is known to get along with (2-eps) times chormatic number colors. I'm going to present some recent progress towards a rigorus explanation of this "hardness transition", which is based in parts on ideas from statistical physics.

Quasirandom Rumor Spreading

Benjamin Doerr, MPI

Randomized rumor spreading is a protocol to disseminate information in networks. A piece of information initially known only to one node is propagated by the following rule: In each round, each node already having the information sends a copy of it to a neighbor chosen uniformly at random.

At SODA 2008, Doerr, Friedrich and Sauerwald suggested a quasirandom analogue. Here, each node has a cyclic list of its neighbors. Once informed, nodes inform their neighbors in the order of the list, starting at a random position. A number of theoretical and practical aspects justify this approach.

While intuitivly clear that the new protocol should be better than theclassical one, giving evidence for this is quite tricky. However, we now have a number of results proving that the new protocol satisfies the same asymptotic performance guarantees as proven for the classical one. Experimental analyses show that the quasirandom model typically is faster and, surpringly, even more robust.

Flipping regular graphs and a peer-to-peer network

Martin Dyer, University of Leeds

Cooper, Dyer and Greenhill showed that random switch moves on d-regular graphs give a rapidly mixing Markov chain with uniform equilibrium distribution. This was used to model the SWAN peer-to-peer network of Holt and Bourassa (2003). Mahlmann and Schindelhauer (2005) proposed the alternative flip chain in the context of peer-to-peer networks.

This was analysed by Feder, Guetz, Mihail and Saberi (2006) using Cooper, Dyer and Greenhill's result and the comparison method for Markov chains. We give a more direct analysis of the flip chain which greatly improves the mixing time estimate. We will also outline the peer-to-peer application.

Joint work with Colin Cooper and Andrew Handley

The Mixing Time of Glauber Dynamics for Colouring Regular Trees

Leslie Goldberg – University of Liverpool

We consider Metropolis Glauber dynamics for sampling proper q-colourings of the n-vertex complete b-ary tree when $3\leq q\leq b/2\ln(b)$. We give both upper and lower bounds on the mixing time. For fixed q and b, our upper bound is $n^{O(b/\log b)}$
and our lower bound is $n^{\Omega(b/q \log(b))}$, where the constants implicit in the $O()$ and $\Omega()$ notation do not depend upon~n, q or~b.

Joint work with Mark Jerrum and Marek Karpinski

A Randomized Algorithm for Metric Bipartite Matching

Anupam Gupta, Carnegie Mellon University

Suppose we have a metric space with k servers and clients arriving online, each client must be irrevocably assigned to some server as soon as it arrives. The cost of an assignment of some clients to servers is given by the sum of the distances between the clients and their assigned servers. How can we minimize this value? While deterministic schemes are Omega(k) competitive for this problem, we show an O(log^2 k) competitive randomized algorithm.

This is joint work with Nikhil Bansal, Niv Buchbinder and Seffi Naor.

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Randomized Algorithms for Metric Constraint Problems

Marek Karpinski, University of Bonn

We design new randomized constant time approximation schemes (CTAS) for metric and quasi-metric CSP problems in a preprocessed model of computation. They entail also the first sublinear time approximation schemes for the above classes of optimization problems.

On the random satisfiable process

Michael Krivelevich, Tel Aviv University

We suggest and analyze a new model for generating random satisfiable k-CNF formulas. To generate such formulas, randomly permute all $2^k\binom{n}{k}$ possible clauses over the variables $x_1,...,x_n$, and starting from the empty formula, go over the clauses one by one, including each new clause as you go along if after its addition the formula remains satisfiable. We study the evolution of this process, namely the distribution over formulas obtained after scanning through the first m clauses (in the random permutation's order).

Our main contribution is as follows. For m>cn, c=c(k) a sufficiently large constant, we are able to characterize the structure of the solution space of a typical formula in this distribution. Specifically, we show that typically all satisfying assignments are essentially clustered in one cluster, and all but $e^{-\Omega(m/n)}n$ of the variables take the same value in all satisfying assignments.

We also describe a polynomial time algorithm that finds with high probability a satisfying assignment for such formulas.

This is a joint work with Benny Sudakov (UCLA) and Dan Vilenchik (Tel Aviv - UC Berkeley).

Is the Web a Socal Graph?

Silvio Lattanzi – Sapienza Università di Roma

Graphs resulting from human behavior (the web graph, friendship graphs, etc.) have hitherto been viewed as a monolithic class of graphs with similar characteristics; for instance, their degree

distributions are markedly heavy-tailed. In this paper we take our understanding of behavioral graphs a step further, by showing that an intriguing empirical property of web graphs --- their compressibility --- cannot be exhibited by well-known social graph models of Kleinberg's \cite{KL00}, by the classic preferential attachment models of Barabasi et al \cite{AB99} and Aiello et al \cite{ACL01}, and by the copying model of Kumar et al \cite{KetAl00}. We then develop a more nuanced model for web graphs and show that it does exhibit compressibility, in addition to previously modeled web graph properties.

Joint work with Flavio Chierichetti, Ravi Kumar, Alessandro Panconesi and Prabhakar Raghavan

Stochastic Online Algorithms

Stefano Leonardi – Sapienza Università di Roma

The performance of an online algorithm (that knows nothing about the future) is usually compared to the optimal solution built with hindsight. This model has led to cleanly defined problems, and strong upper and lower bounds on the competitive ratio are known for most problems of interest.

There are, however, shortcomings to using competitive analysis: the biggest objection being that the strict definition of competitive ratio does not allow us to make fine-grained distinctions between algorithms. Over the years, these drawbacks to the competitive analysis framework have caused researchers to try and weaken the rigid competitive analysis framework, and return to variants of the fundamental question: Can we do better if we are given access to the input distribution? We answer this question in the affirmative for Steiner tree and Set cover problems.

We show that if each demand vertex of a Steiner tree problem is an independent draw from some probability distribution, a variant of the natural greedy algorithm achieves constant competitiveness in the ratio of the algorithm solution to the optimal solution. This compares against a $\Theta(\log n)$ bound known for online Steiner tree algorithms compared against a worst case adversary. This result can be extended to some other subadditive problems as well. Furthermore, we show that both assumptions that the input sequence consists of independent draws from the distribution and that the distribution is known to the algorithm are both essential; there are logarithmic lower bounds if either assumption is violated.

We also apply this kind of analysis to universal set cover problems. Given an universe U of n elements and a weighted collection S of m sets, the universal set cover problem is to a-priori map every element x from U to a set of S containing x, so that for every set X chosen from U the cost of the a-priori solution is as close as possible to the optimal set-cover. For universal mapping, if the set is adversarially chosen, the cost of the a-priori solution can be $\Omega(\sqrt{n})$ times larger than optimal (Jia et al. [STOC'05]). We study the performance of universal set-cover algorithms when the set X of elements to be covered is randomly chosen from the universe U. We compare the expected cost of the a-priori mapping against the expected optimal cost. We present a polynomial-time $O(\log mn)$-competitive universal algorithm. We show similar guarantees for (non-metric) facility location and other covering problems.

These universal mappings naturally give us stochastic online algorithms with the same competitive factors; this complements the $O(\log m \log n)$ worst-case online algorithm for set cover problems of Alon et al. (STOC'03}.

Partitioning Graphs into Balanced Components

Seffi Naor - Computer Science Dept., Technion

We consider the {\em k-balanced partitioning} problem, where the goal is to partition the vertices of an input graph G into k equally sized components, while minimizing the total weight of the edges connecting different components. We allow k to be part of the input and denote the cardinality of the vertex set by n. This problem is a natural and important generalization of well-known graph partitioning problems, including {\em minimum bisection} and {\em minimum balanced cut}.

We present a (bi-criteria) approximation algorithm achievingan approximation of $O(\sqrt{\log{n}\log{k}})$, which matches or improves over previous algorithms for all relevant values of k. Our algorithm uses a semidefinite relaxation which combines $\ell _2^2$ metrics with spreading metrics. Surprisingly, we show that the integrality gap of the semidefinite relaxation is $\Omega (\log{k})$ even for large values of k (e.g., $k=n^{\Omega(1)}$), implying that the dependence on k of the approximation factor is necessary. This is in contrast to previous approximation algorithms for k-balanced partitioning, which are based on linear programming relaxations and their approximation factor is independent of k.

Joint work with Robi Krauthgamer and Roy Schwartz.

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Connectivity and Diameter of Bluetooth-Based Ad Hoc Networks

Andrea Pietracaprina, University of Padua

Consider as a set of n nodes (devices) randomly and uniformly distributed in a square of unit side. Each node has a visibility range of $r(n)$ and, for a given function $c(n)$, it selects as neighbors $c(n)$ visible nodes at random, picking all visible nodes if their number is less than $c(n)$. Such a graph models, for example, the so called Bluetooth topology resulting from the device discovery step in the Bluetooth network formation protocol. In this talk, we will present upper bounds on the minimum $c(n)$, as a function of $r(n)$, which guarantees connectivity, with high probability. Also, we show that when setting $c(n)$ to the established bound, the diameter of the graph is $O(1/r(n) + \ln n)$, with high probability. Finally, we will present some preliminary results on the expansion properties of the graph.

Results are from joint works with P. Crescenzi, C. Nocentini, A. Pettarin, and G. Pucci.

Memoryless Rules for Achlioptas Processes

Oleg Pikhurko - CMU

In an Achlioptas process two random pairs of $[n]$ arrive in each round and the player has to choose one of them. We study the very restrictive version where player's decisions cannot depend on the previous history and only one vertex from the two random edges is revealed.

We prove that the player can create a giant component in (2\sqrt5-4+o(1))n=(0.4721\dots+o(1))n rounds and this is best possible. On the other hand, if the player wants to delay the appearence of a giant, then the optimal bound is $(1/2+o(1))n$, the same as in the Erd\H os-R\'enyi model.

This is joint work with Andrew Beveridge, Tom Bohman, and Alan Frieze.

A Jamming-Resistant MAC Protocol for Single-Hop Wireless Networks

Christian Scheideler - Technische Universität München

In my talk I will consider the problem of designing a medium access control (MAC) protocol for single-hop wireless networks that is provably robust against adaptive adversarial jamming. The wireless network consists of a set of honest and reliable nodes that are within the transmission range of each other. In addition to these nodes there is an adversary. The adversary may know the protocol and its entire history and use this knowledge to jam the wireless channel at will at any time. It is allowed to jam a (1-epsilon)-fraction of the time steps, for an arbitrary constant epsilon>0, but it has to make a jamming decision before it knows the actions of the nodes at the current step. The nodes cannot distinguish between the adversarial jamming or a collision of two or more messages that are sent at the same time. I will demonstrate that there is a local-control MAC protocol requiring only very limited knowledge about the adversary and the network that achieves a constant throughput for the non-jammed time steps under any adversarial strategy above. The protocol is also very energy efficient and it can be extended to obtain a robust and efficient protocol for leader election and the fair use of the wireless channel.

This is joint work with Baruch Awerbuch and Andrea Richa.

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The Multi-Armed Bandit Meets the Web Surfer

Eli Upfal, Brown University

The multi-armed bandit paradigm has been studied extensively for over 50 years in Operations Research, Economics and Computer Science literature, modeling online decisions under uncertainty in a setting in which an agent simultaneously attempts to acquire new knowledge and to optimize its decisions based on the existing knowledge. In this talk I'll discuss several new results motivated by web applications, such as content matching (matching advertising to page contest and user's profile) and efficient web crawling.

On the chromatic number of random d-regular graphs.

Nick Wormald, University of Waterloo

Achlioptas and Moore have announced a proof that random d-regular graphs asymptotically almost surely (a.a.s.) have chromatic number k-1, k, or k+1 where k is the smallest integer satisfying d < 2(k-1)\log(k-1). For about half the values of d, they showed it was k or k+1. We have shown that a.a.s. it is not k+1, which determines the chromatic number a.a.s. for about half the values of d. The proof applies the small subgraph conditioning method to the number of balanced k-colourings, where a colouring is balanced if the number of vertices of each colour is equal, and makes essential use of some of the earlier work of Achlioptas and Naor.

This is joint work with Graeme Kemkes and Xavier Perez.