On Bijective k-Trees Encoding

Saverio Caminiti * Emanuele G. Fusco * Rossella Petreschi *

The problem of coding labeled trees has been widely studied in the literature and several bijective codes that realize associations between labeled trees and sequences of labels have been presented. Among several possible generalizations of trees, k-trees are one of the most natural and interesting. Moreover there is considerable interest in developing efficient tools to manipulate this class of graphs, since many NP-Complete Problems (e.g. Vertex Cover, Graph k-Colorability, Independent Set, Hamiltonian Circuit, etc.) have been shown to be polynomially solvable on k-trees and partial k-trees.

In 1970 A. Rényi and C. Rényi generalized the well known Prüfer code for labeled trees to a subset of labeled k-trees (called Rényi k-trees). Subsequently, a non redundant code that realize bijection between k-trees and a well defined set of code strings has been presented by Ö. Eğecioğlu and L.P. Shen (1988). Unfortunately this bijective code involves a rather difficult computation on the graph and does not seem to admit any efficient implementation.

The bijective code proposed by W.Y.C. Chen (1993), even though admits efficient implementation, encodes only Rényi k-trees and is not suitable for all k-trees. Finally L. Markenzon *et al.* (2005) proposed linear time algorithm for encoding k-trees but the code they handle is not bijective.

In our work we introduce a new bijective code for labeled k-trees. This result rely on the transformation of a given k-trees, through several steps, into a doubly labeled tree (a tree with labels on both nodes and edges). To the best of our knowledge, this is the first code for k-tree shown to admit linear time encoding and decoding algorithms.

*Computer Science Department, *Sapienza* University of Rome, Via Salaria 113, 00198 Roma, Italy. E-mail: {caminiti, fusco, petreschi}@di.uniroma1.it.