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Local dependency dynamic programming in the presence of memory faults

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Memory fault

- One or more bits is read differently from how were last written
- Due to
  - Hardware problems
  - Transient electronic noises
- Impact
  - Machine crash
  - Unpredictable output
  - Security vulnerability

How common are memory errors?

- Cluster of 1000 computers
- 4 GB memory each
- One bit error every **3 seconds!**
- Each computer: 1 error every 50 minutes

[Schroeder, Pinheiro, and Weber. *SIGMETRICS 2009*]

Possible Solutions

- Hardware: ECC (not always available)

- Software: **robustification**
  - Redesign algorithms
  - Rewrite software
  - Faults $\Rightarrow$ longer execution

Faulty RAM model

- Based on the unit cost RAM model
- Adversary
  - Unbounded computational power
  - Can corrupt up to δ words (at any time)
- $O(1)$ *safe* memory words
- $O(1)$ *private* memory words (random bits)

Known results: searching, sorting, dictionaries, priority queues, …

[Finocchi, Italiano, STOC'04]

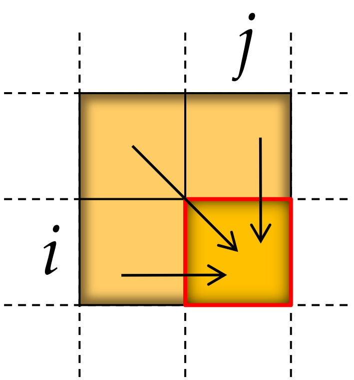
Local dependency dynamic programming

- Strings $X = x_1 \cdots x_n$ and $Y = y_1 \cdots y_m$ $(n \geq m)$
- ED(X, Y) = the number of edit op {ins, del, sub} required to transform X into Y

$$e_{i,j} = \begin{cases} e_{i-1,j-1} & \text{if } x_i = y_j \\ 1 + \min\{e_{i-1,j}, e_{i,j-1}, e_{i-1,j-1}\} & \text{otherwise} \end{cases}$$

- $e_{n,m}$ = ED(X, Y)
- $O(nm)$ running time

DP table



A naïf approach

- Resilient variables
  - Write $2\delta+1$ copies
  - Read by majority (in $O(1)$ safe memory)
- Naïf algorithm $O(nm\delta)$ running time
- Match $O(nm)$ running time of the standard non-resilient implementation $\Rightarrow$ $\delta = O(1)$

Algorithm RED (Resilient Edit Distance)

- Assume X and Y are stored resiliently
- ED(X, Y) can be computed:
  - in $O(nm + \alpha\delta^2)$ time

    $\alpha \leq \delta$ is the actual number of faults
  - correctly w.h.p.

- Assume $m = \Theta(n)$:

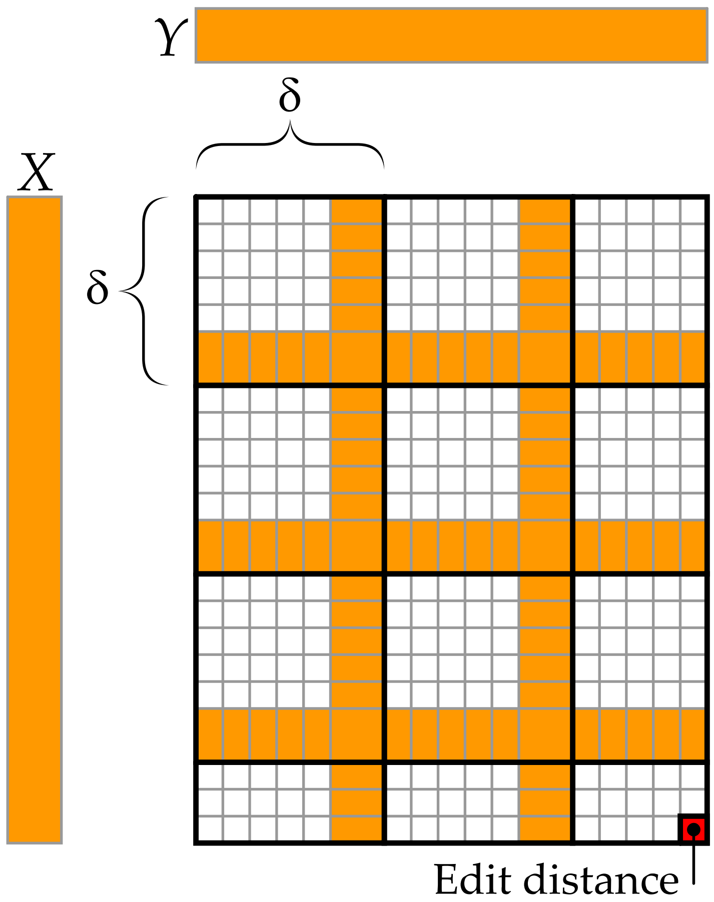
  match $O(n^2)$ $\Rightarrow$ $\delta = O(n^{2/3})$

Techniques

- Resilient variables
- Table decomposition (one-level/hierarchical)
- Karp-Rabin fingerprints
  - Can be computed incrementally in $O(1)$ private memory
- Partial recomputation upon fault detection

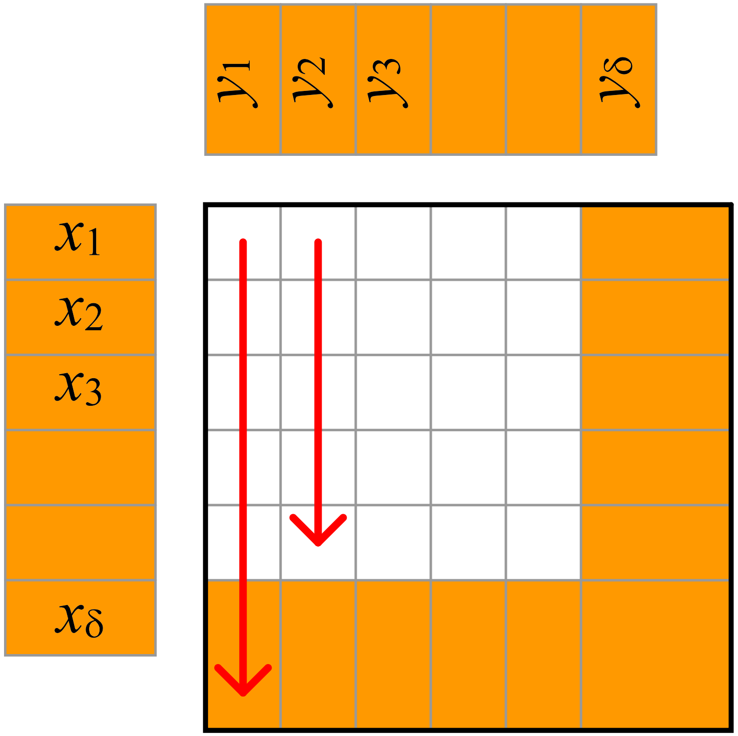
Table decomposition

- DP table is split into blocks of $\delta \times \delta$ cells
- Last row and column are written reliably in the unreliable memory



Block computation

- Column-major order
- While writing column h compute write fingerprint φ_h on written data
- While reading column h compute read fingerprint γ_h on read data
- Fingerprint test: if $\varphi_h \neq \gamma_h$ recompute block
- Similar fingerprints for X and Y

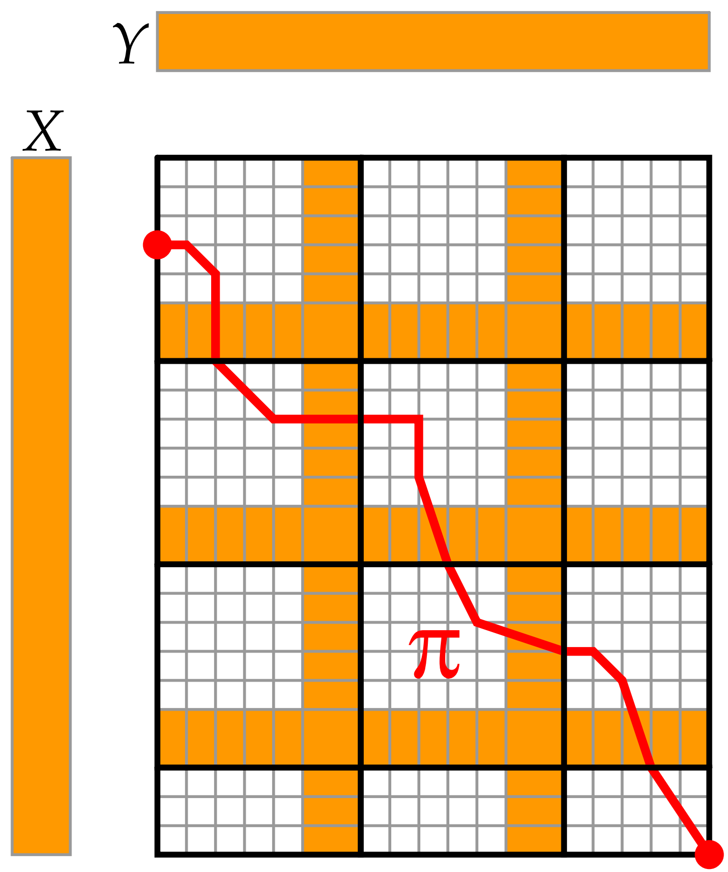


Running time analysis

- Successful block computations:
  - No fingerprint mismatch
  - $O(1)$ amortized cost per operation $\Rightarrow$ $O(nm)$
- Unsuccessful block computations:
  - Each block recomputation can be attributed to (at least) a distinct fault
  - α faults $\Rightarrow$ $O(\alpha\delta^2)$
- Overall running time: $O(nm + \alpha\delta^2)$
- Correct w.h.p. (game based proof)

Tracing back

- Edit sequence is given by π
- In each block traversed by π
  - Compute a segment of π unreliably
  - Verify the segment reading input and block borders reliably
  - Segment not valid $\Rightarrow$ recompute the block forward



Faster error recovery

- Edit distance and sequence can be computed:
  - in $O(nm + \alpha\delta^{1+\varepsilon})$ time
  - correctly w.h.p.

- Assume $m = \Theta(n)$:

$$\text{match } O(n^2) \quad \Rightarrow \quad \delta = O(n^{2/(2+\varepsilon)})$$

Semi-resilient data

- An *r–resilient* variable
  - written in $2r+1$ copies and read by majority
  - can be corrupted (as $r < \delta$) but at the cost of $> r$ faults

- *k* resiliency levels (k constant $= 1/\varepsilon$)
  - level $i \in [1,k]$ uses on δ_i –resilient variables, $\delta_i = \lceil \delta^{i/k} \rceil$

E.g., with $k = 3$

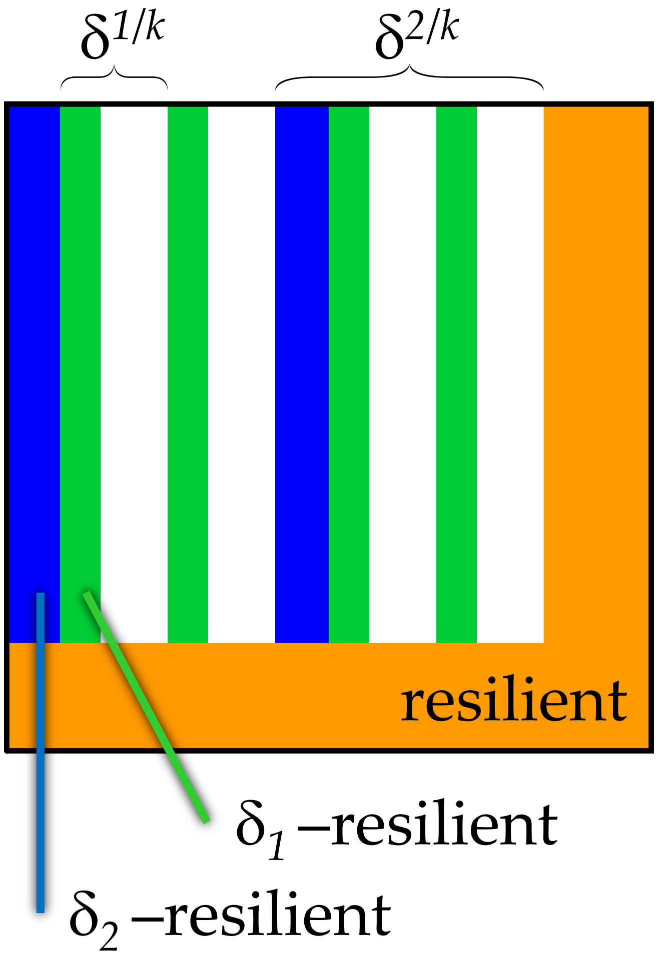
$\delta^{1/3}$ –resilient

$\delta^{2/3}$ –resilient

δ–resilient

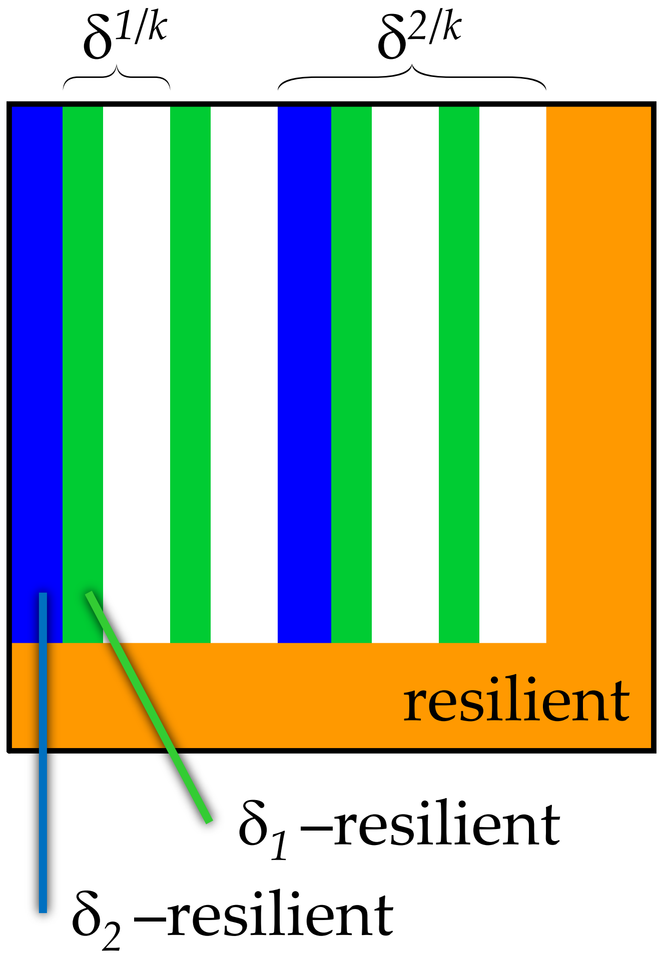
Long-distance fingerprints

- Every δ_i columns we store a δ_i–resilient copy
- One fingerprint for resiliency level (k fingerprints)
- Level i fingerprint associated with the last column written δ_i–resilient



Long-distance fingerprints

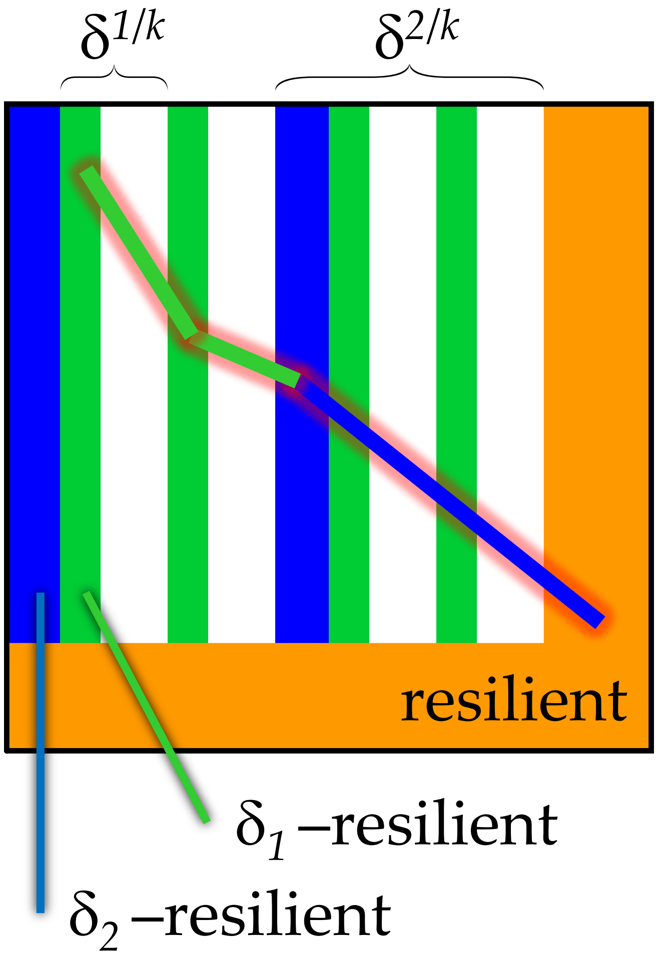
- Fingerprint mismatch on non resilient columns:
  - restart computation from the last δ_1–resilient column
- Fingerprint mismatch while reading at level *i*:
  - restart computation from the last δ_{i+1}–resilient column



Trace-back with semi-resilient cols

- Exploit semi-resilient columns but intermediate fingerprints are no longer available

- Compute segments at resiliency level *i* and glue them together to obtain segments at level *i+1*

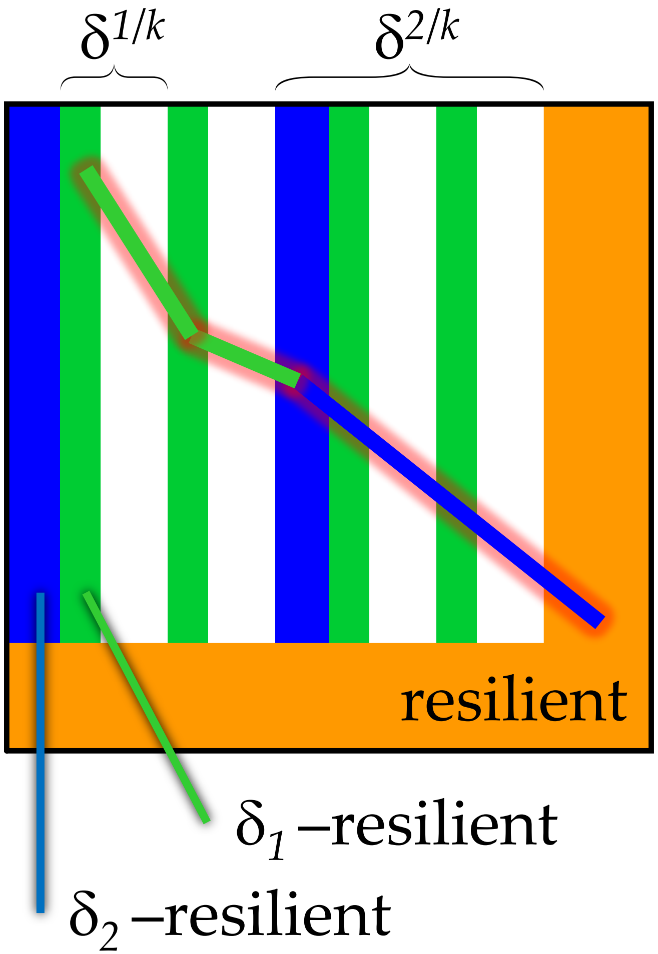


Trace-back with semi-resilient cols

- Level *i* segments are verified against δ_i–resilient columns

- Invalid segment ⇒ recompute forward only the $\delta^{i/k}$ slice of the DP table

$$O(nm + \alpha\delta^{1+\varepsilon})$$



Conclusions

- All Local Dependency Dynamic Programming problems
- Generalize to higher dimensions
- Well known optimization techniques:
  - Hirschberg: reduce space usage
  - Ukkonen: reduce running time if strings are similar

The End